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Data Base Semantics Based on Intuitionistic Logics.

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Introduction.

How semantics of data base is composed ? Semantics of the relational data base model is composed of basic relations in relational algebra. Relational algebra is the set of relations which have a sort of algebraic structure. We can regard it as a sort of category(2). Topos is the category which has a one to one correspondence to higher order intuitionistic logics(3). In this paper, we construct a topos with sets of tuples. Suppose a relation is a set of tuple. Then, by a one to one correspondence to higher order intuitionistic logics, a relation corresponds to a predicate in higher order intuitionistic logics. Therefore, we can regard semantics of the relational data base model as predicate calculus in higher order intuitionistic logics.

As semantics of natural language, Montague semantics is well known. In this, semantics of a sentence is translated into a formula in intensional logics(4) by the Principle of Compositionality. When we ignore intentional operators(4), we can identify formulas in intentional logics with those in higher order intuitionistic logics. Therefore, a part of semantics of natural language corresponds to predicate calculus in higher order intuitionistic logics, namely semantics of the relational data base model, by topos. Also, we can make the correspondence from a part of semantics of the relational data base model to semantics of natural language, by topos.

In this paper, we try to construct topos with sets of tuples so that it is possible to make the above correspondence.

Definitions.

Our aim in this section is to define tuples and relations. At first, we define symbols as follows.

1) Sequence converter.

Let $(t_1,\ldots,t_n)$ be a list. The sequence of $(t_1,\ldots,t_n)$ is defined as $t_1,\ldots,t_n$ and denoted by $\overline{(t_1,\ldots,t_n)}$.

2) Length of a list, index list.

Let $(t_1,\ldots,t_n)$ be a list. The length of $(t_1,\ldots,t_n)$ is defined as n and denoted by $\mathrm{len}((t_1,\ldots,t_n))$. Let ord(n) be {m; m is a natural number and $m \leq n$}. We define index lists of $(t_1,\ldots,t_n)$ as sequences that are composed of elements in $\mathrm{ord}(\mathrm{len}((t_1,\ldots,t_n)))$ without using the same element twice. Namely, if α is a list $(i_1,\ldots,i_n)$, $1 \leq i_j \leq \mathrm{len}((t_1,\ldots,t_n))$, $j=1,\ldots,m$ and $i_1,\ldots,i_m$ are disjoint, α is a index list of $(t_1,\ldots,t_n)$. Let α be an index list of a list t. We define $\tilde{\alpha}$ as an index list that is composed of elements in ord(len(t)) in ascending order except those of α.

3) Projection.

Let t be a list. Let α be an index list of t such that $\alpha = (i_1,\ldots,i_m)$. The projection of t with α is defined as $(t_{i_1},\ldots,t_{i_m})$ and denoted by $t|\alpha$.

4) Selection set.

We define {T,F} as the select set. S denotes it.

5) Alphabet set.

Σ denotes an alphabet set.

We define tuples, type of them and relations, types of them as follows.

1) Let a be an element of Σ^+. Let s be an element of S. (a) and (s) are tuples of type (1) and (0), respectively.

2) Let t_1 and t_2 be tuples of type r_1 and r_2, respectively. $(\bar{t}_1,\bar{t}_2)$ is a tuple of type $(\bar{r}_1,\bar{r}_2)$. (t_1,t_2) is a tuple of type (r_1,r_2).

3) Let R be a set composed of tuples of type t. R is a relation of type t.

4) Let R be a relation of type t. If there is an index list α of t such that R gives the mapping

$$\{r|\alpha;\ r \in R\} \longrightarrow \{r|\tilde{\alpha};\ r \in R\},$$

then (R) is a tuple of type $t|\tilde{\alpha}^{t|\alpha}$.

We define D_t as the set of all tuples whose types are t. Especially, Ω denotes $D_{(0)}$. Let R and α are a relation of type t and an index list of t. $R{\downarrow}\alpha$ denotes $\{r|\alpha;\ r \in R\}$.

Topos.

Let Re be a couple of objects and morphisms which are defined as follows.

1) Object.

Relations are objects.

2) Morphism.

Let R_1 and R_2 be objects of type t_1 and type t_2. If there is an object R so that (R) is a tuple of type $t_1{}^{t}2$ and there exists an index list α so that

$$R|\alpha = R_1 , \quad R|\tilde{\alpha} = R_2 ,$$

then (R) is a morphism.

Since a morphism is a mapping, we define the composition of morphisms as the same of mappings. Then, the associative law is satisfied in these compositions and there exists an identity mapping. Therefore, Re is a category.

In a category Re, $f:A \longrightarrow B$ or $A \xrightarrow{f} B$ denotes a morphism from a object A to a object B. Especially, an identity mapping from A to A is denoted by $id:A \longrightarrow A$ or $A \xrightarrow{id} A$, and an inclusion mapping from A to B (st $A \subseteqq B$) be done by $i:A \longrightarrow B$ or $A \xrightarrow{i} B$.[5] And for each tuple (F) of type $t_1{}^{t}2$, A morphism $f:A \longrightarrow B$ denotes (F) and for each $a \in A$, f(a) denotes a value correspondent to a with f.

Next, we prove that Re is a topos.

Lemma 1. Let R_1 and R_2 be objects of Re. A product of R_1 and R_2 exists in Re.

Proof. Let A and B be objects of Re. Let $A \times B$ be the following relation.

$$\{\ (\bar{a},\bar{b});\ a \in A,\ b \in B\ \}.$$

Then, the following relations Π_1 and Π_2 are subsets of $R_1 \times R_2 \times R_1$ and $R_1 \times R_2 \times R_2$, respectively. (Π_1) and (Π_2) are morphism $\pi_1 : R_1 \times R_2 \longrightarrow R_1$ and $\pi_2 : R_1 \times R_2 \longrightarrow R_2$, respectively.

$$\Pi_1 = \{\ (\bar{r}_1, \bar{r}_2, \bar{r}_1);\ r_1 \in R_1,\ r_2 \in R_2\ \},$$

$$\Pi_2 = \{\ (\bar{r}_1, \bar{r}_2, \bar{r}_2);\ r_1 \in R_1,\ r_2 \in R_2\ \}.$$

Suppose there are an object R and morphisms $f : R \longrightarrow R_1$ and $g : R \longrightarrow R_2$. Then, the following relation $<F,G>$ is a subset of $R \times R_1 \times R_2$ and $(<F,G>)$ is a morphism $<f,g> : R \longrightarrow R_1 \times R_2$.

$$<F,G> = \{\ (\bar{r}, \overline{f(r)}, \overline{g(r)});\ r \in R\ \}.$$

For each element $r \in R$,

$$\pi_1 \circ <f,g>(r) = f(r),$$

$$\pi_2 \circ <f,g>(r) = g(r).$$

Therefore, $\pi_1 \circ <f,g> = f$, $\pi_2 \circ <f,g> = g$

and the following diagram is commutative.

$$\begin{tikzcd}
 & R_1 \\
R \arrow[ur, "f"] \arrow[r, "<f,g>"] \arrow[dr, "g"'] & R_1 \times R_2 \arrow[u, "\pi_1"'] \arrow[d, "\pi_2"] \\
 & R_2
\end{tikzcd}$$

Suppose there is another morphism $k : R \longrightarrow R_1 \times R_2$ such that

$$\pi_1 \circ k = f \quad \text{and} \quad \pi_2 \circ k = g.$$

Then, since for each $r \in R$,

$$<\pi_1 \circ k, \pi_2 \circ k>(r) = <\pi_1 \circ k(r), \pi_2 \circ k(r)>$$

$$= k(r),$$

$$<\pi_1 \circ k, \pi_2 \circ k> = k.$$

Since $\langle \pi_1 \circ k, \pi_2 \circ k \rangle = \langle f,g \rangle$, $\quad k = \langle f,g \rangle$.

Therefore, $R_1 \times R_2$ is a product of R_1 and R_2. QED.

Lemma 2. Re has a terminal object.

Proof. Let R be a relation. $R \times \{(T)\}$ is the unique morphism from R to $\{(T)\}$. Therefore, $\{(T)\}$ is a terminal object.

Lemma 3. Let A and B be objects of Re. A power object of B with A exists in Re.

Prrof. Let E and F be any objects of Re. Let E^F be the following relation.

$$\{(\{(\overline{r},\overline{f(r)});\ r\varepsilon A\});\ f \text{ is a morphism } A \longrightarrow B\}.$$

Let $(-)^A$ be the functor satisfying the following conditions.

i) For each object C of Re, $(-)^A(C) = C^A$.

ii) For each morphism $g;C \longrightarrow D$ of Re, $(-)^A(f)$ is morphism $C^A \longrightarrow D^A$, and

$$(-)^A(g)((\{(\overline{r},\overline{f(r)});\ r\varepsilon A\})) = \{(\overline{r},\overline{g \circ f(r)});\ r\varepsilon A\}$$

for each element $(\{(\overline{r},\overline{f(r)});r\varepsilon A\})$ of C^A.

By Takeuchi's (3), if $(-)^A$ is a right adjoint functor of $(-)\times A$, B^A is a power object. And if the following is provable, $(-)^A$ is a right adjoint functor of $(-)\times A$.

There exists a natural isomorphism, namely

$$\begin{tikzcd} Re(B_1 \times A, C_1) \arrow[rr, "\Theta_{B_1C_1}", "\Theta^{-1}_{B_1C_1}"'] \arrow[d, "F_1"] & & Re(B_1, C_1{}^A) \arrow[d, "F_2"] \\ Re(B_2 \times A, C_2) \arrow[rr, "\Theta_{B_2C_2}", "\Theta^{-1}_{B_2C_2}"'] & & Re(B_2, C_2{}^A) \end{tikzcd}$$

is commutative

where $\Theta^{-1}_{B_iC_i}$ is an inverse mapping of $\Theta_{B_iC_i}$, $i = 1,2$ and F_1 and F_2 are mappings such that for morphisms $f_c:C_1 \longrightarrow C_2$ and $f_B:B_1 \longrightarrow B_2$, and elements $g \varepsilon Re(B_1 \times A, C_1)$ and $h \varepsilon Re(B_1, C_1^A)$,

$$F_1(g) = f_C \circ g \circ (-) \times A(f_B) \qquad \text{and}$$
$$F_2(h) = (-)^A (f_C) \circ h \circ f_B \; .$$

Suppose for each element $g \varepsilon Re(B \times A, C)$ and each element $b \varepsilon B$

$$\Theta_{B_i C_i}(g)(\overline{b}) = (\{(\overline{r}, \overline{g(b,r)}); r \varepsilon A\}) \; , \; i = 1,2.$$

Since $g(b,r) \varepsilon C$ for elements $r \varepsilon A$ and $b \varepsilon B$, $(\{(\overline{r}, \overline{g(b,r)}); r \varepsilon A\})$ is an element of C^A. From the above definition,

$$F_1 \circ \Theta_{B_1 C_1}(g)(b) = (\{(\overline{r}, \overline{(f_{C_1} \circ g(f_{B_1}(b)), r)}); r \varepsilon A\}) \qquad \text{and}$$
$$\Theta_{B_2 C_2} \circ F_2(g)(b) = (\{(\overline{r}, \overline{(f_{C_2} \circ g \circ (-) \times A(f_{B_2})(b,r)}); r \varepsilon A\})$$
$$= (\{(\overline{r}, \overline{(f_{C_2} \circ g \circ (f_{B_2}(b), r)}); r \varepsilon A\}),$$

for each element $b \varepsilon B_2$. Therefore, the above diagram is commutative as to $\Theta_{B_1 C_1}$ and $\Theta_{B_2 C_2}$.

Suppose $\Theta^{-1}_{B_i C_i}(g)((\overline{b}\;\overline{a})) = f(a)$ and $g(b) = (\{(\overline{a}, \overline{f(a)}); a \varepsilon A\})$ for each element $g \varepsilon Re(B_i, C_i^A)$, $b \varepsilon B_i$, $a \varepsilon A$, $i = 1,2$. Then,

$$\Theta_{B_i C_i}(g)((\overline{b}, \overline{a})) = g(b)(a)$$
$$= f(a) \qquad \text{and}$$

$$\Theta_{B_i C_i}(g)(b)(a) = g(b)(a) \; , \; i = 1,2 \quad \text{for each } a \varepsilon A.$$

From the above definition,

$$F_1 \circ \Theta^{-1}_{B_1 C_1}(g)((\overline{b}, \overline{a})) = f_C \circ \Theta^{-1}_{B_1 C_1}(g) \circ (-) \times A(f_B)((\overline{b}, \overline{a}))$$
$$= f_C \circ \Theta^{-1}_{B_1 C_1}(g) \circ ((f_B(\overline{b}), \overline{a})) \; .$$

Suppose $g(f_B(\overline{b}))$ be $(\{(\overline{a}, \overline{f'(a)}); a \varepsilon A\})$. Then,

$$F_1 \circ \Theta^{-1}_{B_1 C_1}(g)((\overline{b}, \overline{a})) = f_C \circ f'(a) \qquad \text{and}$$
$$\Theta^{-1}_{B_2 C_2} \circ F_2(g)((\overline{b}, \overline{a})) = \Theta^{-1}_{B_2 C_2} \circ (-)^A (f_C) \circ g \circ f_B((\overline{b}, \overline{a}))$$
$$= \Theta^{-1}_{B_2 C_2} \circ (-)^A (f_C) \circ g((f_B(b), \overline{a}))$$
$$= f_C \circ f'(a) \; .$$

Therefore, the above diagram is commutative. QED.

Lemma 4. Re has a subobject classifier[3].

Proof. By Takeuchi's[3], if a morphism t: $\{(T)\} \longrightarrow \Omega$ satisfies the following conditions, t is a subject classifier.

(1) For each morphism f: A $\longrightarrow \Omega$ of Re, there exist morphisms g, h and an object B such that

(D1)
$$\begin{tikzcd} B \arrow[r, "h"] \arrow[d, "g"'] & A \arrow[d, "f"] \\ \{(T)\} \arrow[r, "t"] & \Omega \end{tikzcd}$$
is pull back.

(2) For each monomorphism f:A' $\longrightarrow$ A, there exists uniquely the morphism g such that

(D2)
$$\begin{tikzcd} A' \arrow[r, "f"] \arrow[d] & A \arrow[d, "g"] \\ \{(T)\} \arrow[r, "t"] & \Omega \end{tikzcd}$$
is pull back.

Suppose t((T)) = (T).

First, we consider the condition (1). Let B be {a; f(a) = (T) }. Then, there exists a relation $\{(\overline{a},f\overline{(a)}); a\varepsilon B\}$ and it is a subset of $\{(\overline{a},g\overline{(a)}); a\varepsilon A\}$. $(\{(\overline{a},f\overline{(a)}); a\varepsilon B\})$ is a monomorphism B $\longrightarrow$ A . Let h be a morphism $(\{(\overline{a},f\overline{(a)}); a\varepsilon B\})$. Since {(T)} is a terminal object, there exists a morphism B $\longrightarrow$ {(T)} . Let g be the morphism. Then, diagram D(1) is commutative.

Suppose there exist an object C and morphisms j and m such that
$$\begin{tikzcd} C \arrow[drr, bend left, "j"] \arrow[dr, bend right, "m"'] & & \\ & B \arrow[r, "h"] \arrow[d, "g"'] & A \arrow[d, "f"] \\ & \{(T)\} \arrow[r] & \Omega \end{tikzcd}$$
is commutative.

Then, $C = \{ c; c\varepsilon C, f\circ j(c) = (T) \}$.

Let $j(C)$ be $\{ j(c);\ c \in C \}$. Then, $j(C) \subseteq \{ a;\ f(a) = (T) \}$ and $j(C) \subseteq B$.

Therefore, j is a morphism $C \longrightarrow B$ and from definitions of h and the terminal object,

$$h \circ j = j \qquad \text{and}$$
$$g \circ j = m \ .$$

Suppose there exists a morphism k such that $h \circ k = j$ and $g \circ k = m$. Then, from the definition of h and the above proof,

$$k = j \ .$$

Therefore, (D1) is pull back.

Next, we consider the condition (2). Let $f(A')$ be $\{ f(a');\ a' \in A' \}$. There exists the relation K_f such that

$$K_f = \{ (\bar{a}', T);\ f(a') \in f(A') \} \cup \{ (\bar{a}', F);\ f(a') \notin f(A') \}.$$

(K_f) is a morphism $\chi_f : A \longrightarrow \Omega$ such that for each $a \in A$,

$$\chi_f(a) = \begin{cases} (T) & \text{if } a \in f(A'), \\ (F) & \text{otherwise.} \end{cases}$$

Let g be χ_f. Then, (D2) is commutative.

Suppose there exist an object B and morphisms m and n such that

$$\begin{tikzcd} B \arrow[rrd, bend left, "n"] \arrow[rdd, bend right, "m"'] & & \\ & A' \arrow[r, "f"] \arrow[d] & A \arrow[d, "g"] \\ & \{(T)\} \arrow[r] & \Omega \end{tikzcd}$$

is commutative.

Let $n(B)$ be $\{ n(b);\ b \in B \}$. Then $n(B) \subseteq f(A')$.

Since f is a monomorphism, f is a into-mapping. Therefore, there exists the relation $\{ (\bar{a}', \bar{a});\ a \in f(A'),\ a' \in A',\ f(a') = a \}$ and it is the into-mapping $f^{-1}: f(A') \longrightarrow A'$ such that for each a $f(A')$

$f(a') = a$ and $f^{-1}(a) = a'$.

Let k be a morphism's composition $f^{-1} \circ n$. Then, $f \circ k = n$. Similarly to (1), if there exists k' such that $h \circ k' = j$ and $g \circ k' = m$, then $k = k'$. Therefore, (D2) is pull back.

Suppose there exists the morphism g' such that (D2) is pull back. Let $A_{g'}$ be $\{ a;\ a \varepsilon A,\ g(a) = (T) \}$. Then,

$$
\begin{tikzcd}
A_{g'} \arrow[dr, dashed, "k"'] \arrow[drr, bend left, "i"] \arrow[ddr, bend right] & & \\
& A' \arrow[r, tail, "f"] \arrow[d] & A \arrow[d, "g"] \\
& \{(T)\} \arrow[r] & \Omega
\end{tikzcd}
$$

is commutative.

From the definition of pull back, there exists only one morphism k such that $f \circ k = i$. Since $f \circ k(A_{g'}) = i(A_{g'}) = A_{g'}$,

$$f(A') \supset A_{g'} .$$

Since $\quad f(A') \subset A_g$,

$$f(A') = A_{g'} .$$

Therefore, $g = \chi_f$. QED.

Therem 1. Re is a topos.

Proof. By Takeuchi's (3), the category which has products of any objects, the terminal object, power objects and a subobject classifier is a topos. QED.

Let $R_1,\ldots,R_n$ be objects of Re. We define $Re(R_1,\ldots,R_n)$ as a couple of the following objects and morphisms.

(1) (Object).

a) $R_1,\ldots,R_n,\phi$ and Ω are objects.

b) Suppose R is an object. Any subsets of tuples in R are objects. Let t and α be a type of R and an index list of t . Then, $R|\alpha$ is an object.

c) Suppose R and U are objects. Then, $R \times U$, $R^{\vee}U$, $R \cap U$ and $R-U$ are objects.

d) Suppose R is an object. If (R) is a morphism of Re, {(R)} is an object.

(2) (Morphism). It is the same as the morphism of Re.

Any morphisms which are used to prove above lemmas satisfy above conditions. Therefore, as to $PRe(R_1,\ldots,R_n)$, above lemmas are provable.

Cor 1. $PRe(R_1,\ldots,R_n)$ is a topos.

Also, $PRe(R_1,\ldots,R_n)$ is a subcategory of Re. Rules of (object) b) and c) of $PRe(R_1,\ldots,R_n)$ are those of operations of relational algebra[(1)]. The rule of (object) d) and (morphism) are not contained in relational algebra[(1)]. Theses rules are necessary for higher order intuitionistic logics, namely topos. In these, as mappings must be relations constructed by rules a), b), c), and d), constructions of relations which are mappings must be explicitly expressed. Therefore, $PRe(R_1,\ldots,R_n)$ is constructed by relations which are generated from base relations $R_1,\ldots,R_n$ by rule d) and operations of relational algebra.

Similarly to $PRe(R_1,\ldots,R_n)$, as to Re, we can find base relations. They are relations D_t for any types t because any objects of Re are those which are generated from them by rules a) d).

Relations and Intuitionistic logics.

Topos here is one of those which are defined in the previous section. By Takeuchi'(3), the relation A which is an object of topos has one to one correspondence to type A predicate set in higher order intuitionistic logics. Let t be a type of relation A. Let $\bar{D}_t$ be the union of all relations which are objects of topos. Then, the following relation K_A is a subset of $\bar{D}_t \times \Omega$.

$$K_A = \{(\bar{r},T);\ r\varepsilon A\} \cup \{(\bar{r},F);\ r\notin A\} .$$

Since $\bar{D}_t$ and $\bar{D}_t \times \Omega$ are objects of topos, K_A is an object of topos. Also, since (K_A) is a mapping $\bar{D}_t \longrightarrow \Omega$, it is a tuple of type 0^t and is an element of an object $\Omega^{\bar{D}}t$ of topos.

Since $\{(T,(K_A))\}$ is a subset of $\{(T)\} \times \Omega^{\bar{D}}t$, it is an object of topos and is a mapping $\{(T)\} \longrightarrow \Omega^{\bar{D}}t$. Therefore, $(\{(T,(K_A))\})$ is a morphism of topos. By correspondence to higher order intuitionistic logics, the morphism corresponds a predicate of type $^{\bar{D}}t$.

Let (G) be any elements of $\Omega^{\bar{D}}t$. (G) is a morphism $g:\bar{D}_t \longrightarrow \Omega$. The following relation K_g is made from g.

$$K_g = \{(\bar{r},T);\ r\varepsilon\bar{D}_t,\ g(r) = (T)\}.$$

Since K_g is a subset of $\bar{D}_t$, it is an object of topos and

$$K_g = (G).$$

Therefore, each element of a relation $\Omega^{\bar{D}}t$ is determined by each relation A of type t. Each predicate of type $\Omega^{\bar{D}}t$ in higher order intuitionistic logics has a one to one correspondence to each relation of type t. Accordingly, the following theorem is provable.

Theorem 2. Each relation which is an object of topos has a one to one correspondence to each predicate in higher order intuitionistic logics.

Relational algebra[1] that Dr, Codd proposed construct relations from the other relations. By the previous section, topos contain relational algebra. We consider here what of relational algebra correspond to hiher order intuitionistic logics.

At first, we translate each operation of relational algebra by our notation and consider the morphism Correspondent to it.

(Relational algebra).

1) (Cartesian product). Let A and B be relations. $A \times B$ is the cartesian product of A and B.

$$A \times B = \{ (\bar{a}, \bar{b}) ;\ a \varepsilon A,\ b \varepsilon B \}.$$

2) (Union, intersection, difference). Let A and B be relations of the same type. The union, the intersection and the difference of A and B are $A \vee B$, $A \wedge B$ and $A - B$, respectively and

$$A \vee B = \{ a ;\ a \varepsilon A \text{ or } a \varepsilon B \},$$

$$A \wedge B = \{ a ;\ a \varepsilon A,\ a \varepsilon B \} \text{ and}$$

$$A - B = \{ a ;\ a \varepsilon A,\ a \notin B \} .$$

3) (Projection). Let A and α be a relation of type t and an index list of t, respectively. The projection of A on α is $A|\alpha$ and $\quad A|\alpha = \{ a|\alpha ;\ a \varepsilon A \}.$

4) (Join). Let $\theta_{\alpha\beta}$ be a predicate on $\bar{D}_{t_\alpha} \times \bar{D}_{t_\beta}$, namely a morphism $\bar{D}_{t_\alpha} \times \bar{D}_{t_\beta} \longrightarrow \Omega$. Let A and B be relations of type t_1 and t_2 ,respectively and do α and β be index list of t_1 and t_2 such that $t_\alpha = t_1|\alpha$ and $t_\beta = t_2|\beta$, respectively. The $\theta_{\alpha\beta}$-join of A

on α with B on β is $A[\alpha\theta_{\alpha\beta}\beta]B$ and

$$A[\alpha\theta_{\alpha\beta}\beta]B = \{(\bar{a},\bar{b});\ a\varepsilon A,\ b\varepsilon B,\ \theta_{\alpha\beta}(\bar{a}\rceil\alpha,\bar{b}\rceil\beta) = (T)\}.$$

5) (Restriction). Let $\theta_{\alpha\beta}$ be a predicate on $\bar{D}_{t_\alpha}\times\bar{D}_{t_\beta}$, namely a morphism $\bar{D}_{t_\alpha}\times\bar{D}_{t_\beta} \longrightarrow \Omega$. Let A, α and β be a relation of type t and index lists of type t such that $t_\alpha = t|\alpha$ and $t_\beta = t|\beta$, respectively. The $\theta_{\alpha\beta}$-restriction of A on α and β is $A[\alpha\theta_{\alpha\beta}\beta]$ and $A[\alpha\theta_{\alpha\beta}\beta] = \{a;\ a\varepsilon A,\ \theta_{\alpha\beta}(\bar{a}\rceil\alpha,\bar{a}\rceil\beta) = (T)\}$.

Since the operation division is constructed by the other operations, we don't consider it here.

The following lemmas make a correspondence from above operations 1), 2) to morphisms and algebraic structures in topos.

Let A be an object of type t of topos. The following relation K_A is a subset of $\bar{D}_t\times\Omega$. Therefore, it is an object of topos.

$$K_A = \{(\bar{a},T);\ a\varepsilon A\}\ \{(\bar{a},F);\ a\varepsilon\bar{D}_t - A\}.$$

Also, (K_A) is the following mapping χ_A, namely morphism.

$\chi_A : \bar{D}_t \longrightarrow \Omega$ such that for each $r\varepsilon\bar{D}_t$, $\begin{cases}\chi_A(r) = (T) & \text{if } r\varepsilon A,\\ \chi_A(r) = (F) & \text{otherwise.}\end{cases}$

By the proof of lemma 4, χ_A is the classifier of the inclusion mapping $i_A : A \longrightarrow \bar{D}_t$ and the following diagram is pull back(3).

$$\begin{tikzcd} A \arrow[r, "i_A"] \arrow[d] & \bar{D}_t \arrow[d, "\chi_A"] \\ \{(T)\} \arrow[r] & \Omega \end{tikzcd}$$

According to theorem 2, χ_A corresponds to the predicate which corresponds to an object A of topos.

Let I and NI be relations $\{(T,T,T)\}$ and $\{(T,F)\}$, respectively. I and NI are subsets of $\{(T)\}\times\Omega\times\Omega$ and $\{(T)\}\times\Omega$, respectively and are objects of topos. Also, (I) and (NI) are morphisms $\{(T)\} \longrightarrow \Omega\times\Omega$ and $\{(T)\}\times\Omega$, respectively. By Takeuchi's (3), there exist classifiers of (I) and (NI)[3], [5]. $\wedge$ and $\neg$ denote classifiers (I) and (NI), respectively.

Let $U\Omega$ be the relation $\{(T,F), (F,T), (T,T)\}$. $U\Omega$ is a subset of $\Omega\times\Omega$ and is an object of topos. There exists the inclusion mapping $i : U\Omega \longrightarrow \Omega\times\Omega$ and it is a morphism of topos. Topos have the classifier of i[3], [5]. $\vee$ denotes it.

By the definition of classifier, each of $\wedge$, $\neg$ and $\vee$ is only one morphism such that each of the following diagrams is pull back.

$$
\begin{tikzcd}
\{(T)\} \arrow[r, "(I)"] \arrow[d] & \Omega\times\Omega \arrow[d, "\wedge"] \\
\{(T)\} \arrow[r] & \Omega
\end{tikzcd}
,\quad
\begin{tikzcd}
\{(T)\} \arrow[r, "(NI)"] \arrow[d] & \Omega\times\Omega \arrow[d, "\neg"] \\
\{(T)\} \arrow[r] & \Omega
\end{tikzcd}
\quad \text{and} \quad
\begin{tikzcd}
U\Omega \arrow[r, "i"] \arrow[d] & \Omega\times\Omega \arrow[d, "\vee"] \\
\{(T)\} \arrow[r] & \Omega
\end{tikzcd}
.
$$

Let A and K_A be an object of type t of topos and the following relation, respectively.

$$K_A = \{(\bar{a},T);\ a\varepsilon A\}\cup\{(\bar{a},F);\ a\varepsilon \bar{D}_t - A\} .$$

Then, K_A is the mapping $\bar{D}_t \longrightarrow \Omega$. χ_A denotes a morphism (K_A).

Lemma 6. Let A and B be objects of topos. Then, $A\times B$ is an object of topos.

Proof. Trivial.

Lemma 7. Let A and B be objects of the same type t of topos.

$$\chi_{A\cap B} = \wedge\circ(\chi_A, \chi_B) ,$$

$$\chi_{A\cup B} = \vee\circ(\chi_A, \chi_B) \text{ and}$$

$$\chi_{A-B} = \wedge\circ(\chi_A, \neg\circ\chi_B) .$$

Proof. 1) Suppose $a \varepsilon A \cap B$. Then,

$$\chi_{A \cap B}(a) = (T) \quad \text{and}$$

$$\wedge^{\circ}(\chi_A(a), \chi_B(a)) = \wedge^{\circ}((T,T)) = (T).$$

Suppose $a \varepsilon \bar{D}_t - A \cap B$. Then,

$$\chi_{A \cap B}(a) = (F) \quad \text{and}$$

$$\wedge^{\circ}(\chi_A(a), \chi_B(a)) = \begin{Bmatrix} \wedge^{\circ}((T,F)) \\ \wedge^{\circ}((F,T)) \\ \wedge^{\circ}((F,F)) \end{Bmatrix} = (F).$$

Therefore, $\chi_{A \cap B} = \wedge^{\circ}(\chi_A, \chi_B)$.

2) Suppose $a \varepsilon A \cup B$. Then,

$$\chi_{A \cup B}(a) = (T) \quad \text{and}$$

$$\vee^{\circ}(\chi_A(a), \chi_B(a)) = \begin{Bmatrix} \vee^{\circ}((T,F)) \\ \vee^{\circ}((F,T)) \end{Bmatrix} = (T).$$

Suppose $a \varepsilon \bar{D}_t - A - B$. Then,

$$\chi_{A \cup B}(a) = (F) \quad \text{and}$$

$$\vee^{\circ}(\chi_A(a), \chi_B(a)) = \vee^{\circ}((F,F)) = (F).$$

Therefore, $\chi_{A \cup B} = \vee^{\circ}(\chi_A, \chi_B)$.

3) Suppose $a \varepsilon A - B$. Then,

$$\chi_{A-B}(a) = (T) \quad \text{and}$$

$$\wedge^{\circ}(\chi_A(a), \neg^{\circ}\chi_B(a)) = \wedge^{\circ}((T,T)) = (T).$$

Suppose $a \varepsilon \bar{D}_t - (A-B)$. Then,

$$\chi_{A-B}(a) = (F) \quad \text{and}$$

$$\wedge^{\circ}(\chi_A(a), \neg^{\circ}\chi_B(a)) = \begin{Bmatrix} \wedge^{\circ}((T,F)) \\ \wedge^{\circ}((F,T)) \\ \wedge^{\circ}((F,F)) \end{Bmatrix} = (F).$$

Therefore, $\chi_{A-B} = \wedge^{\circ}(\chi_A, \neg^{\circ}\chi_B)$. QED

Lemma 8. Let A and α be an object of type t of topos and an index list of t. Then, there exists the morphism such that $A \longrightarrow A|\alpha$.

Proof. Since $\{(\bar{a},\bar{a}\rceil\alpha);\ a\varepsilon A\}$ is a subset of $A\times A|\alpha$, it is an object of topos. Also, the relation is the mapping $A \longrightarrow A|\alpha$. QED.

Let A and B be objects of type t_1 and t_2 of topos, respectively. Let α and β be index list of t_1 and t_2, respectively. Let Θ be the following.

$$\{(\bar{a}\rceil\alpha,\bar{b}\rceil\beta,T);\ a\varepsilon A,\ b\varepsilon B,\ \theta_{\alpha\beta}(\bar{a}\rceil\alpha,\bar{b}\rceil\beta) = (T)\}$$
$$\{(\bar{a}\rceil\alpha,\bar{b}\rceil\beta,F);\ a\varepsilon A,\ b\varepsilon B,\ \theta_{\alpha\beta}(\bar{a}\rceil\alpha,\bar{b}\rceil\beta) = (F)\}\ .$$

Then, Θ is a subset of $A|\alpha\times B|\beta\times\Omega$ and is an object of topos. (Θ) is a morphism of topos $A|\alpha\times B|\beta \longrightarrow \Omega$. θ_T denotes this morphism.

By definitions, $A[\alpha\Theta_{\alpha\beta}\beta]B$ is a subset of $A\times B$.

Therefore, $A[\alpha\theta_{\alpha\beta}\beta]B$ is an object of topos and there exists the morphism $i : A[\alpha\theta_{\alpha\beta}\beta]B \longrightarrow A\times B$ in our topos.

Lemma 9. The following diagram is pull back.

$$\text{(D1)}\quad \begin{array}{ccc} A[\alpha\theta_{\alpha\beta}\beta]B & \xrightarrow{\ i\ } & A\times B \\ \downarrow & & \downarrow \theta_T \\ \{(T)\} & \longrightarrow & \Omega \end{array}\ .$$

Especially, suppose $\theta_{\alpha\beta}$ is "=" and morphisms $P_A : A[\alpha=\beta]B \longrightarrow A$, $P_B : A[\alpha=\beta]B \longrightarrow B$, $P_\alpha : A \longrightarrow A|\alpha\times B|\beta$ and $P_\beta : B \longrightarrow A|\alpha\times B|\beta$ are $(\{(\bar{a},\bar{b},\bar{a});\ (\bar{a},\bar{b})\varepsilon A[\alpha=\beta]B\})$, $(\{(\bar{a},\bar{b},\bar{b});\ (\bar{a},\bar{b})\varepsilon A[\alpha=\beta]B\})$, $(\{(\bar{a},\bar{a}\rceil\alpha);\ a\varepsilon A\})$ and $(\{(\bar{b},\bar{b}\rceil\beta);\ b\varepsilon B\})$, respectively. Then, since the type of $A|\alpha$ is the same type of $B|\beta$, morphisms P_A, P_B, P_α and P_β are those of our topos and make the following diagram be pull back.

(D2)
$$\begin{tikzcd} A[\alpha=\beta]B \arrow[r, "P_A"] \arrow[d, "P_B"'] & A \arrow[d, "P_\alpha"] \\ B \arrow[r, "P_\beta"] & A|\alpha\times B|\beta \end{tikzcd}$$
.

Proof. 1) The proof of (D1). By definitions of i and θ_T, this diagram is commutative. Suppose there exist an object C and morphisms m and n such that the following diagram is commutative.

$$\begin{tikzcd} C \arrow[drr, bend left, "n"] \arrow[dr, bend right, "m"'] & & \\ & A[\alpha\theta_{\alpha\beta}\beta]B \arrow[r, "i"] \arrow[d] & A\times B \arrow[d, "\theta_T"] \\ & \{(T)\} \arrow[r] & \Omega \end{tikzcd}$$
.

Let $i(A[\alpha\theta_{\alpha\beta}\beta]B)$ be $\{ i((\overline{a},\overline{b}));\ (\overline{a},\overline{b})\varepsilon A[\alpha\theta_{\alpha\beta}\beta]B \}$. Then, since i is an into mapping, there exists an inverse mapping of i i^{-1}: $i(A[\alpha\theta_{\alpha\beta}\beta]B) \longrightarrow A[\alpha\theta_{\alpha\beta}\beta]B$ in our topos. Similarly to the proof of lemma 4, there exists only one morphism k such that

$i^\circ k = n$.

2) The proof of (D2). By Goldbatt's (5), it is trivial. QED.

Let A, α and β be an object of type t of our topos and index list of t. Let Θ be the following.

$\{(\overline{a},T);\ a\varepsilon A,\ \theta_{\alpha\beta}((\overline{a}\rceil\alpha,\overline{a}\rceil\beta)) = (T)\}\cup\{(\overline{a},F);\ a\varepsilon A,\ \theta_{\alpha\beta}((\overline{a}\rceil\alpha,\overline{a}\rceil\beta)) = (F)\}$.

Then, Since Θ is a subset of $A\times\Omega$, it is an object in our topos and (Θ) is a morphism $A \longrightarrow \Omega$ in our topos. θ_T denotes this morphism.

Lemmma 10. The following diagram is pull back.

$$\begin{tikzcd} A[\alpha\theta_{\alpha\beta}\beta]B \arrow[r, "i"] \arrow[d] & A \arrow[d, "\theta_T"] \\ \{(T)\} \arrow[r] & \Omega \end{tikzcd}$$
.

Proof. Similarly to the previous lemma, it is provable. QED.

By lemma 6 $\sim$ 10. Each operation of relational algebra corresponds to a morphism or an algebraic structure of our topos. Next, by taking theses correspondences, we make correspondences from operations of relational algebra to higher order intuitionistic logics.

(1) (Product). Let A and B be objects of type t_1 and t_2 of our topos, respectively. Similarly to lemma 7, $\chi_{A\times B} = \vee^\circ(\chi_A, \chi_B)$ is provable. Therefore, by theorem 2 and the coorespondence(3) from our topos to higher order intuitionistic logics, if predicates correspondent to χ_A and χ_B are $P_A(a)$ and $P_B(b)$, respectively. $\chi_{A\times B}$ corresponds to

$$P_A(a) \vee P_B(b) \quad .$$

(2) (Union, intersection, difference). By lemma 7 and the correspondence(3), (5) from our topos to higher order intuitionistic logics, if predicates correspondent to χ_A and χ_B are $P_A(a)$ and $P_B(a)$, respectively, $\chi_{A\vee B}$, $\chi_{A\wedge B}$ and χ_{A-B} correspond to

$P_A(a) \vee P_B(a)$, $P_A(a) \wedge P_B(a)$ and $P_A(a) \wedge \neg P_B(a)$,respectively.

(3) (Projection). By Takeuchi's (3), we define a morphism $(\exists A)f : Y \longrightarrow \Omega$ for each morphism $f : A\times Y \longrightarrow \Omega$, in our topos. Let $F_A : A \longrightarrow \Omega$ be $(\{(\bar{a},F) : a \in A\})$. Then, F_A is a morphism of our topos and for each $a \in A$, $F_A(a) = (F)$. By the definition of topos, there exists the morphism $\ulcorner F_A \urcorner : Y \longrightarrow \Omega^A$ (3) such that the following diagram is commutative.

$$\begin{tikzcd}
A\times Y \arrow[d, "id\times\ulcorner F_A\urcorner"'] \arrow[dr, "F_A\times Y"] & \\
A\times\Omega^A \arrow[r, "ev"] & \Omega
\end{tikzcd}$$

In Takeuchi's (3) if f and g are morphism $A \longrightarrow B$ and $A \longrightarrow C$, respectively,

$$<f,g> = (\{(f(\bar{a}),g(\bar{a}));\ a\varepsilon A\}).$$

We define $(\exists A)f$ as follows.

$$(\exists A)f = \neg\circ(=_\Omega A)\circ<\hat{f},\ulcorner F_A\urcorner>^{(3)}.$$

By Takeuchi's (3), $(=_\Omega A)\circ<\hat{f},\ulcorner F_A\urcorner>$ is a morphism of our topos. Therefore, $(\exists A)f$ is a morphism of our topos.

In Takeuchi's (3), $(\forall A)f$ (3) corresponds to the universal quantifier and the existential quantifier is constructed by $\forall$, -> and $\wedge$. In topos, this composition of $\forall$, -> and $\wedge$ coincides with $(\exists A)f$. Therefore, we can make $(\exists A)f$ correspond to the existential quantifier.

Let A and α be an object of type t of our topos and an index sequence of t. Then, the following lemma is provable.

Lemma 11.

$$\chi_{A|\alpha} = (\exists\bar{D}_t|\tilde{\alpha})\chi_A .$$

Proof. Suppose $a|\alpha\varepsilon A|\alpha$. By Takeuchi's (3),

$$\begin{aligned}(\bar{D}_t|\alpha)\circ\chi_A(a|\alpha) &= \neg\circ(=_\Omega\bar{D}_t|\tilde{\alpha})\circ<\hat{\chi}_A,\ulcorner F_{\bar{D}_t|\tilde{\alpha}}\urcorner>(a|\alpha)\\ &= \neg\circ(=_\Omega\bar{D}_t|\tilde{\alpha})\circ(\chi_A,\ulcorner F_{\bar{D}_t|\tilde{\alpha}})\\ &= (T) \quad .\end{aligned}$$

Suppose $a|\alpha\varepsilon\bar{D}_t|\alpha - A$. By Takeuchi's (3),

$$\begin{aligned}(\bar{D}_t|\alpha)\circ\chi_A(a|\alpha) &= \neg\circ(=_\Omega\bar{D}_t|\tilde{\alpha})\circ<\hat{\chi}_A,\ulcorner F_{\bar{D}_t|\tilde{\alpha}}\urcorner>(a|\alpha)\\ &= \neg\circ(=_\Omega\bar{D}_t|\tilde{\alpha})\circ(F_{\bar{D}_t|\alpha},F_{\bar{D}_t|\tilde{\alpha}})\\ &= (F) \quad .\end{aligned}$$

QED.

If the predicate correspondent to χ_A is $P_A(a)$, (Projection) $\chi_{A|\alpha}$ corresponds to

$$\exists\cdot xP_A((x,a))^{(3)}.$$

4) (Join).

Lemma 13.

$$\chi_{A[\alpha\theta_{\alpha\beta}\beta]B} = \wedge^\circ\langle\wedge^\circ(\chi_A,\chi_B),\theta_{\alpha\beta}\rangle \ .$$

Proof. Suppose $a\varepsilon A$ and $b\varepsilon B$.

$$\chi_{A[\alpha\theta_{\alpha\beta}\beta]B}((\bar{a},\bar{b})) = \theta_{\alpha\beta}((\overline{a}\rceil\alpha,\overline{b}\rceil\beta)) \qquad \text{and}$$

$$\wedge^\circ\langle\wedge^\circ(\chi_A,\chi_B),\theta_{\alpha\beta}\rangle((\bar{a},\bar{b})) = \wedge^\circ(\wedge^\circ((T,T)),\theta_{\alpha\beta}((\overline{a}\rceil\alpha,\overline{b}\rceil\beta))$$
$$= \theta_{\alpha\beta}((\overline{a}\rceil\alpha,\overline{b}\rceil\beta)) \ .$$

Suppose $a\notin A$ or $b\notin B$.

$$\chi_{A[\alpha\theta_{\alpha\beta}\beta]B}((\bar{a},\bar{b})) = (F) \qquad \text{and}$$

$$\wedge^\circ\langle\wedge^\circ(\chi_A,\chi_B),\theta_{\alpha\beta}\rangle((\bar{a},\bar{b})) = \wedge^\circ(F,\theta_{\alpha\beta}((\overline{a}\rceil\alpha,\overline{b}\rceil\beta)))$$
$$= (F) \ . \qquad \text{QED.}$$

If predicates correspondent to χ_A, χ_B and $\theta_{\alpha\beta}$ are $P_A(a)$, $P_B(a)$ and $P_\theta((a|\alpha,b|\beta))$, respectively. $\chi_{A[\alpha\theta_{\alpha\beta}\beta]B}$ corresponds to

$$P_A(a)\wedge P_B(b)\wedge P_\theta((\overline{a}\rceil\alpha,\overline{b}\rceil\beta)) \ .$$

5) (Restriction). Similarly to lemma 13,

$$\chi_{A[\alpha\theta_{\alpha\beta}\beta]} = \wedge^\circ\langle\chi_A,\theta_{\alpha\beta}\rangle \qquad \text{is provable.}$$

If predicates correspondent to χ_A and $\theta_{\alpha\beta}$ are P (a) and $P_\theta((\overline{a}\rceil\alpha,\overline{b}\rceil\beta))$, respectively. $\chi_{A[\alpha\theta_{\alpha\beta}\beta]}$ corresponds to

$$P_A(a)\wedge P_\theta((\overline{a}\rceil\alpha,\overline{a}\rceil\beta)) \ .$$

Predicate of 1) $\sim$ 2) satisfy the following conditions in higher order intuitionistic logics.

$$P_A(a),\ P_B(b) \ \vdash \ P_A(a)\vee P_B(b) \ ,$$
$$P_A(a),\ P_B(a) \ \vdash \ P_A(a)\vee P_B(a) \ ,$$
$$P_A(a),\ P_B(a) \ \vdash \ P_A(a)\wedge P_B(a) \ ,$$
$$P_A(a),\ \neg P_B(a) \ \vdash \ P_A(a)\wedge\neg P_B(a) \ ,$$
$$P_A((\overline{a}\rceil\alpha,\overline{a}\rceil\beta)) \ \vdash \ \exists\cdot xP_A((\overline{a}\rceil\alpha,x)) \ ,$$

$$P_A(a),\ P_B(b),\ P_\theta((\overline{a}\rceil\alpha,\overline{b}\rceil\beta))\ \vdash\ P_A(a)\wedge P_B(b)\wedge P_\theta((\overline{a}\rceil\alpha,\overline{b}\rceil\beta)),$$

and

$$P_A(a),\ P_\theta((\overline{a}\rceil\alpha,\overline{a}\rceil\beta))\ \vdash\ P_A(a)\wedge P_\theta((\overline{a}\rceil\alpha,\overline{a}\rceil\beta)) \quad .$$

In above reductions, if pemises are satisfied, we can gain operations of 1) $\sim$ 5). Namely, relatioal algebra is expressed by above reductions.

Let s be the following term in higher order intuitionistic logics.

$$\{\ P_1,\ldots,P_n,Q\ \mid [U(P_1,\ldots,P_n) \rightarrow V(P_1,\ldots,P_n)]\wedge Q = V\ \}^{(3)}.$$

By Takeuchi's (3),

$$\vdash \exists\cdot s\ .$$

Let $A_1,\ldots,A_n$ and B be types of $x_1,\ldots,x_n$ and y. If

$$\vdash \forall\cdot x_1\ldots x_n \exists\cdot!y\ s(x_1,\ldots,x_n,y) \text{ ----}(\alpha),$$

there exists the morphism $A_1\times\ldots\times A_n \longrightarrow B$ (3). Above reductions satisfy (α) when U and V are the premise and the conclusion, respectively. Since the quantifier $\exists\cdot!y$ means that there exists a certain function, this function is the conclusion $V(P_1,\ldots,P_n)$, namely each operation of relational algebra.

Besides above reductions, we can many sorts of reductions and show that for each reduction, there exists the function (3). Therefore, by taking reductions, we can consider constructions of relations, namely semantics of relational data bases.

According to the Principle of Compositionality in Montague semantics, the expression of "~ of ~" is the following term [3].

$$\{ P,Q \mid Q(a)\wedge \exists\cdot x(Q(x)\wedge P(a,x))\} .$$

By taking this term, we can express "parent of parent" as follows.

$$\{ P,Q \mid Q(a)\wedge \exists\cdot x(Q(a)\wedge P(a,x))\} (FM, \exists\cdot zFM(b,z))$$
$$= \exists\cdot zFM(a,z)\wedge \exists\cdot x(\exists\cdot zFM(x,z)\wedge FM(a,x)) .$$

By previous sections, we can abstract operations from reductions which are made in the higher order intuitionistic logics correspondent to PRe(R). Namely, from the reduction

$$FM(b,a) \quad \vdash \exists\cdot xFM(b,x) ,$$

we can gain the relation $R|\alpha$.

"name of parent" and "parent of parent" are conclusion of the following reductions.

$$N(a), FM(a,b), N(b) \vdash N(a)\wedge \exists\cdot x(N(x)\wedge FM(a,x)) \qquad \text{and}$$
$$FM(a,b), FM(b,c) \vdash \exists\cdot zFM(c,z)\wedge \exists\cdot x(\exists\cdot zFM(x,z)\wedge FM(c,x))$$

These reductions are constructed by those of (intersection), (join) and (projection) in previous section. Therefore, we can gain relations correspondent to "name of parent" and "parent of parent" after using these operations on $R|\alpha$ $R|\tilde{\alpha}$ and $R|\tilde{\alpha}$ according to each correspondent reduction in the previous section.

By taking the composition of each attribute of R and "~ of ~" , we can construct the new relation from R whose attributes are "name of parent" and "parent of parent" [2].

$\{ P,Q \mid Q \text{ of } P \}$ denotes $\{P,Q \mid Q(a)\wedge \exists\cdot x(Q(x)\wedge P(a,x))\}$. In higher order intuitionistic logics, this is a term of type $_{\Omega}\Omega^{\bar{D}}(1)\times\Omega^{\bar{D}}(1)^{\times\bar{D}}(1)$ and the following reduction is provable.

Relation and natural language.

In this section, we use Montague semantics as semantics of natural language and consider it in higher order intuitionistic logics. Formulas which are considered here correspond to objects of topos, namely relations. By this correspondence, we can introduce semantics of natural language to that of relational data bases, namely constructions of relations by operations. Here, as an example, we consider semantics of "~ of ~".

Now, we consider the following relational data base R.

R	name	parent
t_1 ;	a_1	b_1
.	.	.
.	.	.
.	.	.
t_n ;	a_n	b_n

Since PRe(R) consturcts the higher order intuitionistic logics whose types are objects of PRe(R) [(3)], we can consider Montague semantics in this logics. In this semantics, attributes of R "name" and "parent" are translated into predicates in this logics. N(a) and P(a) denote predicates which express "name" and "parent" respectively. Semantics domains of N(a) and P(a) are $R|\alpha^{\vee}R|\tilde{\alpha}$ and $R|\tilde{\alpha}$, respectively. Therefore, types of N(a) and P(a) are $\Omega^{R|\alpha^{\vee}R|\tilde{\alpha}}$ and $\Omega^{R|\tilde{\alpha}}$, respectively. By taking a predicate FM(a,b) such that a is a "parent" of b in R, P(a) is expressed by $\exists \cdot x FM(a,b)$ and the type of it is $\Omega^{R|\alpha^{\vee}R|\tilde{\alpha}\times R|\alpha^{\vee}R|\tilde{\alpha}}$.

By taking the above, the sentence in natural language "name of parent" is expressed as follows.

$$N(a)\wedge \exists \cdot x(N(x)\wedge FM(a,x)).$$

N(a) FM(a,b), N(b) FM(b,c) $\vdash$ N of FM (b) FM of FM (c) . This is constructed by reductions of N(a) and FM(a,b), namely reductions of 1) $\sim$ 5) in the previous section. Therefore, this reduction show, how to make the relation whose attributes are "name of parent" and "parent of parent" from R.

In stead of N and FM, for any predicate of type Ω^t and $\Omega^{t\times t}$, this reduction is provable. Therefore, if predicates of these types P_1(a) and $P_2((\bar{a},\bar{b}))$ express attributes of any relation R, we can gain the relation whose attributes are expressed by P_1 of P_2 (a) and P_2 of P_2 $((\bar{a},\bar{b}))$ from R and show how to construct it. In this case, we can construct it by operations of relational algebra (1).

By taking the above way, we can introduce a part of semantics of natural language to those of relational data base.

Topos in this paper have not a correspondence to a intensional operators (4) which is used in Montague semantics. Therefore, we can't introduce all of Montague semantics to those of relational data bases in this model.

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